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THE MUTUAL INTERACTION OF MULTIPLE VORTEXES AND ITS INFLUENCE ON BINARY AND SINGLE TROPICAL VORTEX SYSTEMS [1]

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ABSTRACT

A two-layer discrete vortex model developed previously is applied to the study of short-term displacements of tropical vortexes due to their mutual interaction. The model treats the tropical cells as point-form vortexes and obtains analytic solutions for the stream functions in two layers in terms of Hankel functions of zero order. These solutions form a system of ordinary differential equations that governs the motion of the vortex filaments of finite strengths in the lower and upper layers. The model can handle a finite number of vortexes simultaneously. It also allows the influence of a basic flow to be considered.

Initially, the model is applied to some analytical data in an attempt to study the patterns of motion for both binary and single tropical vortex systems. It is found that the individual vortexes exhibit a variety of complex trajectories depending upon their strengths and tilts, and upon stability parameters, μ_1^2 and μ_2^2. Of interest in this study is the "self-interaction" concept according to which the upper level circulation of a sloping vortex interacts with the lower level circulation. It is shown that the observed short-period oscillations of the surface trajectory of a tropical cyclone can be explained to a reasonable extent using this concept. A case study is made involving multiple vortexes over the North Atlantic Ocean. The results illustrate how mutual interaction influences the motion of individual vortexes and further demonstrate the importance of the surrounding vortexes in modifying the well-known cyclonic rotation of a binary system.

Preliminary calculations indicate that the model may provide a useful tool for predicting short-term displacements of tropical cyclones.

1. INTRODUCTION

In the vast oceanic regions of tropical latitudes, one often finds two or more closed circulations, some of which are of tropical cyclone intensity, existing simultaneously in close proximity to one another. The movement of these tropical cells is influenced by their mutual interaction as well as by a steering current in which they may be embedded. Observations indicate that, when a pair of tropical cyclones exist simultaneously in close proximity, they generally move around each other in a cyclonic sense when the steering flow is weak. This cyclonic rotation of the binary tropical cyclones has been commonly known as the "Fujiwhara effect" after his studies on vortical systems of the atmosphere (Fujiwhara 1923). For a binary vortex system, the rate of rotation can be calculated by using an expression given by Lamb (1945). Using this expression, Haurwitz (1951) studied the motion of some binary tropical cyclones of the North Atlantic Ocean; more recently Brand (1970) has made similar calculations for some binary tropical cyclones of the Western North Pacific Ocean. Riehl (1954) discussed a synoptic example of the interaction between a tropical vortex pair and indicated that external large-scale vortexes commonly present on synoptic maps may also influence the motion of the pair. Thus, when more than two vortexes are present in close proximity, one must use a general expression to calculate the influence of mutual interaction. Shabbar (1968), with a primary interest in extended range forecasting, developed a two-layer atmospheric model in which the continuous vorticity distribution is replaced by point-form vortexes. Details of this model can be found in Shabbar (1968) and Shabbar and Khandekar (1970); it is sufficient to say here that, utilizing this model, a trajectory calculation of a vortex could easily be made, taking into account its interactions with surrounding vortexes.

Observational studies on the motion of tropical cyclones have revealed that a detailed trajectory of a hurricane vortex exhibits a small and rather irregular oscillatory motion. From a detailed study of two hurricane tracks, Horn (1951) obtained a sinusoidal motion with periods of 20 and 40 hr; Senn (1961), making use of a radar film, demonstrated oscillatory and irregular motion of hurricane centers with a period from half an hour to 1 hr or more. Although theoretical studies by Yeh (1950) indicate oscillatory motion of a tropical cyclone with a period of 2 to 3½ days, it is not clear whether the above-mentioned shorter periods can be explianed by a mechanism similar to that put forward by Yeh.

In this investigation, a two-layer discrete vortex model is utilized to study the patterns of vortex motion which are generated by mutual interaction. Making use of analytical data, we have studied the motions of binary and single tropical vortex systems both with and without a basic flow. It is found that the individual vortexes exhibit a variety of complex trajectories depending on their strengths, tilts, and stability parameters. A vortex having a sloping

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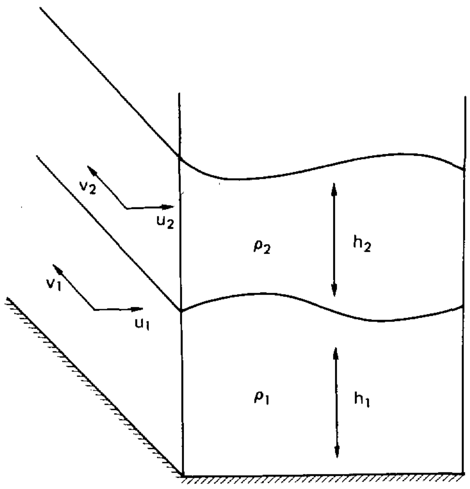

FIGURE 1.—The two-layer model used in this study.

axis is represented in the model by two distinct vortex columns as discussed in section 3; this enables the upper part of the vortex to interact with the lower part, producing "self-interaction." It is further shown that such a tilted vortex, when embedded in a uniform easterly current moves in a trochoidal path with a period from 1 hr to several hours. Finally, we present a case study involving multiple vortexes over the North Atlantic with a view to exploring the possibility of applying these results to forecasting short-term displacements of tropical cyclones.

2. THE MODEL

Figure 1 illustrates an atmosphere with two layers of incompressible, homogeneous, and inviscid fluid with a free surface. Let u, v, h, and ρ be the two horizontal wind components, the depth, and the density in each layer, respectively. Further, let subscripts 1 and 2 signify the lower and upper layers, respectively.

For each layer, the continuity and momentum equations can be written in terms of u, v, h, and ρ. Consider the entire motion as a disturbance of the state of equilibrium with equilibrium depths h_1^0 and h_2^0. We express, following Shabbar (1968), the dependent variables u, v, and h in a power series expansion in terms of the Froude number, F, with the restraint that $F=U^2/gH \ll 1$ (typical magnitudes of U and H pertain to large-scale motions, namely, $U=10$ m/s and $H=10$ km). Equate the coefficients of like powers of F and, after some manipulations (Shabbar and Khandekar 1970), the first-order vorticity equations for the lower and upper layers are obtained as

$$\frac{d_1^{(1)}}{dt}[\nabla^2\psi_1-\mu_1^2(\psi_1-\psi_2)]=0 \tag{1}$$

and

$$\frac{d_2^{(1)}}{dt}[\nabla^2\psi_2+\mu_2^2(\psi_1-\psi_2)]=0. \tag{2}$$

Here,

$$\frac{d_{1,2}^{(1)}}{dt}=\frac{\partial}{\partial t}+u_{1,2}^{(1)}\frac{\partial}{\partial x}+v_{1,2}^{(1)}\frac{\partial}{\partial y}.$$

The superscript (1) refers to the order of approximation, ψ_1 and ψ_2 are the stream functions for the two layers, $\mu_1^2=f^2/gh_1^0(\Delta\rho/\rho_1)$ and $\mu_2^2=f^2/gh_2^0(\Delta\rho/\rho_1)$. In these expressions, f is the Coriolis parameter, its variations in the y direction being ignored; $\Delta\rho$ is the density difference between the lower and the upper layer and $\Delta\rho/\rho_1 \ll 1$ for this study.

If one utilizes eq (1) and (2), the movement of a finite number of geostrophic point vortexes can be studied in an unbounded x-y plane. For this purpose we assume that the expressions $\nabla^2\psi_1-\mu_1^2(\psi_1-\psi_2)$ and $\nabla^2\psi_2+\mu_2^2(\psi_1-\psi_2)$ are zero everywhere except in small isolated regions that are approximated by point singularities. Thus, the continuously distributed potential vorticity field is replaced by point-form vortexes. The spirit of the procedure is similar to that adopted in classical hydrodynamics (Lamb 1945). Morikawa (1962) in his work on prediction of hurricane tracks also used a similar concept of a geostrophic point vortex.

Now imagine that there are n vortexes present both in the lower and the upper layer. Let the positions of n vortexes in the lower layer be given by (x_k, y_k) and in the upper layer by (X_k, Y_k) with k running from 1 through n. Then we have

$$\nabla^2\psi_1-\mu_1^2(\psi_1-\psi_2)=\sum_{k=1}^{n}\Gamma_1^k\delta(|r-r_k|) \tag{3}$$

for the lower layer and

$$\nabla^2\psi_2+\mu_2^2(\psi_1-\psi_2)=\sum_{k=1}^{n}\Gamma_2^k\delta(|r-R_k|) \tag{4}$$

for the upper layer.

Here, $\delta(|r-r_k|)$ and $\delta(|r-R_k|)$ are Dirac δ-functions, Γ is the strength of a vortex,

$$(r-r_k)^2=(x-x_k)^2+(y-y_k)^2$$

and

$$(r-R_k)^2=(x-X_k)^2+(y-Y_k)^2.$$

After some manipulations which are outlined in the appendix, the stream functions may be obtained as

$$\begin{aligned}\psi_1=&-\frac{\mu_1^2}{4(\mu_1^2+\mu_2^2)}\left[\sum_{k=1}^{n}\Gamma_1^k iH_0^{(1)}(i\sqrt{\mu_1^2+\mu_2^2}|r-r_k|)\right.\\&\left.-\sum_{k=1}^{n}\Gamma_2^k iH_0^{(1)}(i\sqrt{\mu_1^2+\mu_2^2}|r-R_k|)\right]\\&+\frac{\mu_2^2}{2\pi(\mu_1^2+\mu_2^2)}\sum_{k=1}^{n}\Gamma_1^k\ln|r-r_k|\\&+\frac{\mu_1^2}{2\pi(\mu_1^2+\mu_2^2)}\sum_{k=1}^{n}\Gamma_2^k\ln|r-R_k|\end{aligned} \tag{5}$$

and

$$\psi_2=\frac{\mu_2^2}{4(\mu_1^2+\mu_2^2)}\left[\sum_{k=1}^{n}\Gamma_1^k iH_0^{(1)}(i\sqrt{\mu_1^2+\mu_2^2}|r-r_k|)\right.$$
$$\left.-\sum_{k=1}^{n}\Gamma_2^k iH_0^{(1)}(i\sqrt{\mu_1^2+\mu_2^2}|r-R_k|)\right]$$
$$+\frac{\mu_2^2}{2\pi(\mu_1^2+\mu_2^2)}\sum_{k=1}^{n}\Gamma_1^k\ln|r-r_k|$$
$$+\frac{\mu_1^2}{2\pi(\mu_1^2+\mu_2^2)}\sum_{k=1}^{n}\Gamma_2^k\ln|r-R_k|. \quad (6)$$

Equations (5) and (6) give the values of the stream functions in the two layers in terms of Hankel functions $H_0^{(1)}$ of zero order, for which the arguments are the coordinates of the vortex positions in the lower and upper layers. Further, when eq (5) and (6) are differentiated with respect to x and y, we obtain the u and v components of a vortex situated at $(x_k/X_k,\ y_k/Y_k)$ at which points the derivatives are evaluated. We thus obtain a system of ordinary differential equations governing the motion of vortexes in the two layers:

$$\left.\frac{dx}{dt}\right|_k=-\left.\frac{\partial\psi_1}{\partial y}\right|_k;\ \left.\frac{dy}{dt}\right|_k=\left.\frac{\partial\psi_1}{\partial x}\right|_k \text{ for the lower layer}$$

and (7)

$$\left.\frac{dX}{dt}\right|_k=-\left.\frac{\partial\psi_2}{\partial Y}\right|_k;\ \left.\frac{dY}{dt}\right|_k=\left.\frac{\partial\psi_2}{\partial X}\right|_k \text{ for the upper layer.}$$

These equations define the vortex motion due entirely to its interaction with other vortexes. Further, the influence of a uniform steering current can easily be taken into account. Consider, for example, that the vortexes are embedded in a uniform northeasterly current, U, at an angle, α, to the north. In this case, the u and v components of the vortex are:

$$\frac{dx}{dt}=-\frac{\partial\psi}{\partial y}-U\sin\alpha;\ \frac{dy}{dt}=\frac{\partial\psi}{\partial x}-U\cos\alpha. \quad (8)$$

Similar equations are obtained for the upper layer.

The total number of ordinary differential equations [eq (7)] will depend upon the number of vortexes in each layer; that is, with two vortexes in each layer we will have a system of eight differential equations (one for each of the two coordinates of the four vortex centers) and they are as follows:

$$\frac{dx_1}{dt}=-\frac{\partial\psi_1}{\partial y_1};\ \frac{dx_2}{dt}=-\frac{\partial\psi_1}{\partial y_2};\ \frac{dy_1}{dt}=\frac{\partial\psi_1}{\partial x_1};\ \frac{dy_2}{dt}=\frac{\partial\psi_1}{dx_2}$$

and (9)

$$\frac{dX_1}{dt}=-\frac{\partial\psi_2}{\partial Y_1};\ \frac{dX_2}{dt}=-\frac{\partial\psi_2}{\partial Y_2};\ \frac{dY_1}{dt}=\frac{\partial\psi_2}{\partial X_1};\ \frac{dY_2}{dt}=\frac{\partial\psi_2}{\partial X_2}.$$

The right side of each of these equations is obtained by differentiating the expressions for the stream functions ψ_1 and ψ_2 and evaluating the same at various points. A typical equation, say for dx_1/dt, will be written as:

$$\frac{dx_1}{dt}=-\frac{\partial\psi_1}{\partial y_1}=-\frac{1}{2\pi\mu^2}\left[\Gamma_1^2\mu_2^2\frac{y_1-y_2}{(r_1-r_2)^2}+\Gamma_2^1\mu_1^2\frac{y_1-Y_1}{(r_1-R_1)^2}\right.$$
$$+\Gamma_2^2\mu_1^2\frac{y_1-Y_2}{(r_1-R_2)^2}-\Gamma_1^2\mu_1^2K_0'(\mu|r_1-r_2|)\mu\frac{y_1-y_2}{|r_1-r_2|}$$
$$\left.+\Gamma_2^1\mu_1^2K_0'(\mu|r_1-R_1|)\mu\frac{y_1-Y_1}{|r_1-R_1|}+\Gamma_2^2\mu_1^2K_0'(\mu|r_1-R_2|)\mu\frac{y_1-Y_2}{|r_1-R_2|}\right]$$

with $\mu^2=\mu_1^2+\mu_2^2$.

Here, we have expressed $H_0^{(1)}(x)$ in terms of $K_0(x)$, the Bessel function of the first kind, by means of the identity $iH_0^{(1)}(ix)=(2/\pi)K_0(x)$. Furthermore, $K_0'(x)$ is the derivative of $K_0(x)$ with respect to its argument. With the right side being known, eq (9) can be integrated with respect to time using a suitable numerical procedure.

In this study, we have used Hamming's (1959) modified predictor-corrector method to solve the system of ordinary differential equations, eq (9). This is a stable fourth-order integration procedure that requires the evaluation of the right side of the system only twice per time step. This is a great advantage compared with other methods of the same order of accuracy, especially the Runge-Kutta method which requires the evaluation of the right side four times per time step. Secondly, at each step the calculation procedure gives an estimate of the local truncation error; thus, the procedure is able to choose and change the step size, Δt, so as to maintain a uniform preassigned tolerance limit throughout the integration period. Further, the predictor-corrector method is not self-starting; that is, the functional values at a single previous point are not enough to get the functional values ahead. Therefore, a special Runge-Kutta method (Ralston 1962) followed by one iteration step is used initially to generate the starting values. Additional details about the numerical procedure may be found in the application program of International Business Machines Corp. (1968).

3. EXPERIMENTS USING ANALYTICAL DATA

Hypothetical data are used to study the patterns of vortex motion in some simple and special cases of both binary and single vortex systems The locations of the vortex centers, the values of the vortex strengths, and the parameters μ_1^2 and μ_2^2 are prescribed in each case. We have assigned some arbitrary values to (x_1, y_1), (X_1, Y_1), etc., so that the pair of vortexes under consideration are separated by a distance of 1500 km in both layers. The vortex strength is estimated by the product of its area and average vorticity. The parameters μ_1^2 and μ_2^2 are evaluated from a knowledge of f, the Coriolis parameter, and representative values of h_1^0, h_2^0, and $\Delta\rho/\rho_1$. As mentioned earlier, variations of f with latitude were neglected and an average value of f representative for 20°N was utilized. The value of $\Delta\rho/\rho_1$ was taken as 10^{-2} and the equilibrium depths h_1^0 and h_2^0 were both assumed to be equal to 5 km. Finally, the set of differential equations was numerically integrated using a maximum time step of 20 min to obtain the following results.

MOTION OF A BINARY VORTEX SYSTEM

We consider first a simple configuration of two cyclonic vortexes of equal strength in each layer placed along an arbitrary x-axis. A value of 5×10^{11} cm^2/s is used for the vortex strength. This value represents a tropical cyclone having a radius of 300 km and an average vorticity of 2×10^{-4}s^{-1}.

Initially, we consider the situation wherein the vortex tubes are stretched out vertically from lower layer to upper layer with no tilt. This is a special case of a barotropic fluid with no density stratification. From hydrodynamical considerations (Lamb 1945) it is well known that the motion of each vortex is entirely due to the other and is always perpendicular to the line joining their centers. Since the two vortexes are of equal strength, they will always remain at the same distance from each other and will rotate with constant angular velocity about the center of the system. The ensuing motion is thus a cyclonic rotation in a circle with constant angular velocity that is directly proportional to the strength of each vortex and inversely proportional to the square of the distance separating them.

In this configuration, the interaction terms between the lower and the upper vortexes do not contribute at any time since the vortex tubes always remain vertical. If, however, a vortex slopes with height, the coordinates (x,y) of the lower layer vortex will be different from those (X,Y) of the upper layer vortex. This enables the upper part of the vortex to interact with the lower part producing "self-interaction." The term self-interaction here refers to the process by which a sloping vortex influences its own trajectory at a particular level by interacting with its circulation at another level. To determine the influence of self-interaction on the surface trajectory of a tropical vortex, we must have a knowledge of the slope of such a vortex. It is difficult to estimate the slope of the axis of a mature tropical cyclone. If the slope were appreciable, many of the present observational studies would have diagnosed it. Mainly due to lack of firm observational evidence, it is commonly assumed that the axis is vertical. This, however, does not preclude the possibility that a slight slope might exist. To the authors' knowledge, Simpson (1947) is one of the earliest to provide some information on the slope of the axis of a tropical cyclone. He conceded that some inaccuracy might exist in the slope he presented. According to him, the axis of the Florida hurricane of 1946 appeared to be truly vertical from the surface to about 4550 m (15,000 ft), showed a tilt slightly to the rear up to 6100 m (20,000 ft), and tilted thereafter in the opposite direction so that the center at 7600 m (25,000 ft) was separated from the surface center by as much as 160 km (100 mi). In this study, we assume the axis of a mature tropical cyclone to be slightly tilted in the upper level as shown (on an exaggerated scale) in figure 2. In this case, the vortex column AB will be represented mathematically by two vertical columns at positions (x_j,y_j) and (X_j,Y_j) in the lower and upper layers, respectively. With such an arrangement, we will have nonzero interactions between the vortexes of the lower and upper layers. In the calculations for the model, the vortex axis is assumed to be tilted in such a way as to separate the vortex centers in the lower and upper layer by a horizontal distance of 50 km. (The actual magnitude of the tilt of the axis is not critical. What is required is the separation of the vortex centers in the two layers by a finite amount.) Further, we have assigned the strength of one vortex to be twice that of the other vortex; that is, $\Gamma_1^1=5\times10^{11}$ and $\Gamma_2^1=1\times10^{12}$. (For the corresponding two vortexes in the upper layer, slightly smaller values of strengths are assigned.) With these modifications, the model is integrated without a basic flow for a period of 24 hr. The displacement of the vortexes in the lower layer is shown in figure 3 (dashed lines). We see that the vor-

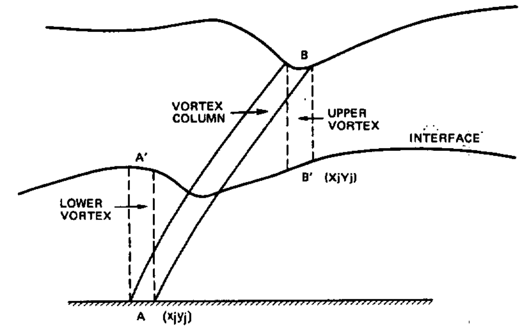

FIGURE 2.—Representation of a vortex column in the two-layer model.

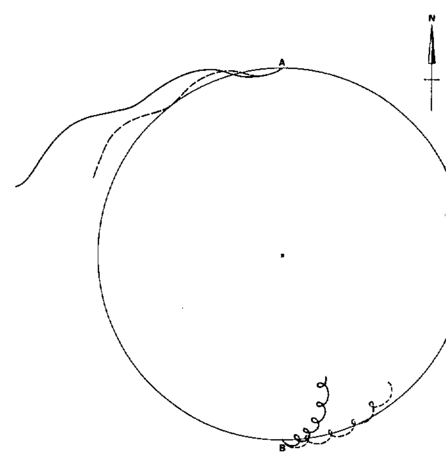

FIGURE 3.—Motion of a binary vortex system with (solid lines) and without (dashed lines) a steering current. The strength of the vortex at B is arbitrarily assigned to be twice that of the vortex at A.

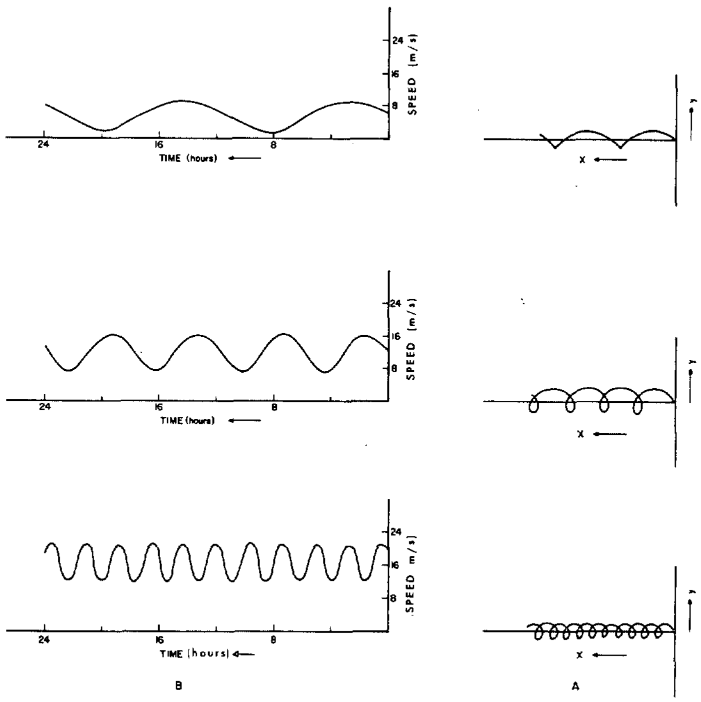

FIGURE 4.—(A) trajectories of vortex motion using various values of strength and tilt; and (B) corresponding speed versus time curves (see table 1 for details).

texes rotate in a cyclonic sense with the weaker one (located at A with $\Gamma_1^1=5\times10^{11}$) moving away from the circular path. Further, the weaker vortex describes a sinusoidal trajectory, while the stronger one (at B) describes a trochoidal path with distinct loops. This oscillatory motion of each vortex is attributed to the tilt in the axis of the vortex as it stretches out from lower layer to upper layer.

As a next step, we consider the pair of vortexes to be embedded in a uniform easterly flow of 3 m/s. As such, we use the set of equations given by eq (8) and integrate them as before to obtain the vortex displacements shown by the solid lines of figure 3. The basic pattern is still the same as before; however, the trajectory of each vortex is modified due to the steering flow. The weaker vortex (at A) appears to move farther away from the circular path. while the stronger one moves in a trochoidal path with the distance between successive loops increasing with time as the influence of the steering flow increases. As the speed of the steering current increases, it is found that the cyclonic rotation of the vortex at B is so much modified that it eventually starts drifting in the direction of the steering flow.

MOTION OF A SINGLE VORTEX EMBEDDED IN A UNIFORM EASTERLY CURRENT

As before, we have assumed the axis of the vortex to be slightly tilted in the upper layer. For the purpose of numerical experiments, we have used a number of different values for the tilt of the vortex as well as for the strength of the vortex. In all cases, the basic pattern of motion is trochoidal in the direction of the steering current. Figure 4A shows three different trajectories of a single vortex obtained for appropriate values of the tilt and the strength

TABLE 1.—*Values of strength and tilt of the vortex column for the trajectories of figure 4*

Trajectory	Tilt	Strength	Period
	(km)	(cm²/s)	(hr)
Top	50	0.5×10^{12}	11.5
Center	100	1.5×10^{12}	6.0
Bottom	50	2.5×10^{12}	2.3

of the vortex. For all three cases, a uniform easterly current of 5 m/s is used. The corresponding speed versus time curves are shown in figure 4B. The top trajectory has a period of 11.5 hr, the center trajectory has a period of 6 hr, while the bottom trajectory has a period of 2.3 hr. The values of the tilt and the strengths used in obtaining these trajectories are given in table 1.

From these and various other experiments conducted in this connection it is found that: (1) the period increases as the tilt of the vortex is increased; (2) the period decreases as the strength of the vortex is increased; (3) the period is not altered by variations in the speed of the steering current, but the distance between loops along the trochoidal path is increased; and finally (4) the increase in μ_1^2 and μ_2^2 (decrease in the static stability) gives rise to an unstable and irregular motion for the vortex center. [The parameters μ_1^2 and μ_2^2 can be increased by either decreasing the static stability, $\Delta\rho/\rho_1$, or by decreasing the equilibrium depths h_1^0 and h_2^0. A detailed discussion regarding the effect of the variation of parameters μ_1^2 and μ_2^2 on vortex motion is given in Shabbar and Khandekar (1970).]

This oscillatory motion of the vortex center appears to be in close agreement with observations of small-scale hurricane movement reported by Horn (1951) and Senn (1961). Periodic looping of a hurricane center has also been reported in some other studies (e.g., Jordan and Stowell 1955). In Horn's study of hurricane tracks, a period of 20 hr and more is suggested, but Senn's study with the help of radar films indicates a period from half an hour to an hour or so. Our experiments with analytical data have indicated that a periodic motion from 1 hr to about 24 hr can be obtained by suitable variations of the strength and the tilt of the vortex. This oscillatory motion of the vortex center appears to be initiated as a result of tilt in the vortex axis through the mechanism of self-interaction. As observed earlier, the interaction terms in eq (9) do not contribute to the movement of a single vortex so long as the axis of the vortex remains vertical from lower layer to upper layer. As soon as a small tilt is introduced in the vortex axis, the upper part of the vortex interacts with the lower part and this, combined with the influence of the steering current, generates trochoidal motion along the direction of the steering current. In his classical work Yeh (1960) considers the influence of a basic flow and the Coriolis force and obtains a period of the order of 2–3½ days for the motion of a hurricane vortex; however, his mechanism seems inadequate to explain the smaller scale motion as suggested by Senn. More recently, Kuo (1969) has studied the motion of vortexes in shear flow and friction in an attempt to explain the motion of rotating thunderstorm cells and tornadoes. According to Kuo, the trochoidal motion is created by the interaction between the circulation of the vortex and the mean current and is superimposed upon the mean path. Our study suggests the mechanism of self-interaction as a means of explaining the trochoidal motion of a large hurricane vortex.

4. EXPERIMENTS USING REAL DATA

To test the applicability of this model, some preliminiary calculations were made using data from the routinely available Northern Hemisphere historic map series for sea level and the 500-mb level to represent the lower and the upper layers, respectively, in the model. The parameters μ_1^2 and μ_2^2 were evaluated using a mean value of f and representative values of h_1^0, h_2^0, and $\Delta\rho/\rho_1$. The value of $\Delta\rho/\rho_1$ was taken as 10^{-2} and the equilibrium depths h_1^0 and h_2^0 were taken as 3 km each. The strength of an individual vortex defined in terms of a representative area and vorticity was estimated subjectively from the map series at both sea level and the 500-mb level. The locations of the vortex centers as given in the Northern Hemisphere historic map series at the two levels were taken as the initial positions of the vortexes. By drawing quasi-circular isolines, the limit of the circulation was determined subjectively for each vortex. An average vorticity field was likewise estimated. The subjective but judicious placing of the centers of the lower and upper vortexes in some instances resulted in imparting slight tilts to the vortexes. Further, the vortex strength was generally found to be smaller at the upper level; the strength was assumed to remain constant throughout the integration period. Several case studies were made to study the displacements of tropical cyclones surrounded by other cyclonic and anticyclonic vortexes. Using a maximum time step of 20 min, predictions were initially made for up to 24 hr in advance and were extended to 48 hr in some cases. We present below the results of a case study involving multiple vortexes over the North Atlantic Ocean to illustrate the typical patterns of tropical cyclone predictive and verifying displacements obtained by this model.

Figure 5 shows schematically the positions of some low- and high-pressure centers over the North Atlantic Ocean for 0000 GMT, Sept. 11, 1961. Only a portion of the North Atlantic Ocean is included on a Mercator projection; for convenience the latitudes and longitudes of the low- and high-pressure centers are also given. Since the main interest in this study is the comparison between observed and predicted displacements for a pair of tropical cyclones, no synoptic maps are presented here. The observed 24-hr displacements of the tropical cyclones, L_1 and L_2, are shown by solid arrows. In the first step, we considered only the interaction between the tropical cyclones L_1 and L_2. The model with two vortexes ($n=2$) in each layer produced the well-known cyclonic rotation as shown by the dashed arrows, F_1. (All the predicted displacements F_1, F_2, etc., are shown by smoothed trajectories.) In the second step, the subtropical high-pressure cells H_1 and H_2 were included in the calculation. The number of vortexes in each layer became four. The resulting displacements of the binary tropical cyclones are shown by the dashed arrows, F_2. It can be readily seen that, by including H_1 and H_2, the model produced improved predictions, especially for the tropical cyclone L_2. In the third step, the model took into account the influence of the tropical cyclone L_3. The number of vortexes now became five and the resulting displacements for L_1 and L_2 are shown by the dashed arrows, F_3. An improved prediction resulted for L_1 and the deviation between the observed and the forecast displacement increased for L_2. Finally, we considered the influence of the low-pressure cell L_4 that was embedded in the westerlies. The model, now using six vortexes in each layer, obtained the predicted displacements for L_1 and L_2 as shown by the dashed arrows, F_4.

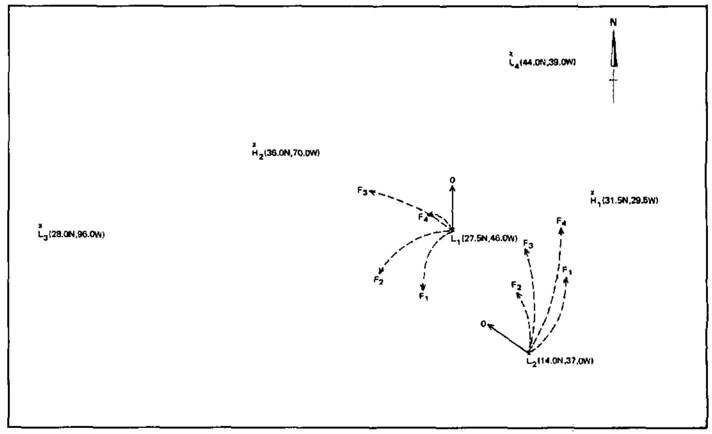

FIGURE 5.—Twenty-four-hr observed (solid arrows) and predicted (dashed arrows) displacements of the tropical cyclones L_1 and L_2. Predicted displacements are obtained after considering the influences of various combinations of surrounding vortexes. See text for details.

These results demonstrate, very generally, how the inclusion of surrounding vortexes influences the predicted movement of a binary vortex system. In particular, it is interesting to note the influence of the subtropical high-pressure cells (H_1 and H_2) and the rapidly moving midlatitude system (L_4) on the predicted movement of the binary system. These results also suggest an explanation as to why the cyclonic rotation of a simple binary tropical vortex system, calculated from elementary hydrodynamical considerations, is often not well observed. From our case studies over the North Atlantic Ocean, we find that the inclusion of the semipermanent high-pressure cells, as well as the midlatitude low-pressure cells, is essential for improved predictions of both binary and single vortex systems.

5. CONCLUDING REMARKS

The short-term displacements of tropical votexes due to mutual interaction was studied with the aid of a two-layer discrete vortex model. Employing some analytical data, we studied the patterns of motion for both binary and single tropical vortex systems. We found that the individual vortexes exhibited a variety of complex trajectories depending on their strengths, slopes of their axes with height, and stability parameters. Of interest in this study was the utilization of a self-interaction concept that enabled the upper part of a tilted vortex to interact with its lower part. The observed short-period oscillations of the surface trajectory of a tropical cyclone could be explained to a reasonable extent using this concept.

Several case studies of the movements of multiple vortexes over the North Atlantic for a period of 24 to 48 hr were made using this model. The results illustrated, among other things, how the well-known cyclonic rotation of a binary vortex system experiences a modification due to the presence of (other) surrounding vortexes.

In this model, we assumed the strengths of the vortexes to remain constant during the forecast period; we also assumed the presence of a uniform steering current. We are currently modifying the equations to include energy sources and sinks and accordingly to let the strengths of the vortexes vary during the forecast period. We also intend to study the effect of shear flow on vortex motion.

APPENDIX: DERIVATION OF STREAM FUNCTIONS

Consider eq (3) and (4) of the text. Subtract eq (4) from eq (3) to obtain:

$$\nabla^2(\psi_1-\psi_2)-(\mu_1^2+\mu_2^2)(\psi_1-\psi_2)=\sum_{k=1}^{n}\Gamma_1^k\delta(|r-r_k|)-\sum_{k=1}^{n}\Gamma_2^k\delta(|r-R_k|).$$

This is a Helmholtz-type equation the solution of which can be written as:

$$\psi_1-\psi_2=-\frac{1}{4}\sum_{k=1}^{n}\Gamma_1^k iH_0^{(1)}(i\sqrt{\mu_1^2+\mu_2^2}|r-r_k|)+\frac{1}{4}\sum_{k=1}^{n}\Gamma_2^k iH_0^{(1)}(i\sqrt{\mu_1^2+\mu_2^2}|r-R_k|). \quad (10)$$

Here, $H_0^{(1)}$ is the Hankel function of zero order. Next, multiply eq (3) by μ_2^2 and eq (4) by μ_1^2 and add to get:

$$\nabla^2(\mu_2^2\psi_1+\mu_1^2\psi_2)=\sum_{k=1}^{n}\Gamma_1^k\mu_2^2\delta(|r-r_k|)+\sum_{k=1}^{n}\Gamma_2^k\mu_1^2\delta(|r-R_k|).$$

This is a Poisson-type equation the solution of which is given by:

$$\mu_2^2\psi_1+\mu_1^2\psi_2=\frac{1}{2\pi}\sum_{k=1}^{n}\Gamma_1^k\mu_2^2\ln|r-r_k|+\frac{1}{2\pi}\sum_{k=1}^{n}\Gamma_2^k\mu_1^2\ln|r-R_k|. \quad (11)$$

From eq (10) and (11), we can write solutions for the stream functions ψ_1 and ψ_2 as given in the text.

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